

# Perturbative Bottom-up Approach for Neutrino Mass Matrix in Light of Large $\theta_{13}$ and Role of Lightest Neutrino Mass

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## Abstract

We discuss the role of lightest neutrino mass ( $m_0$ ) in the neutrino mass matrix, defined in a flavor basis, through a *bottom-up* approach using the current neutrino oscillation data. We find that if  $m_0 < 10^{-3}\text{eV}$ , then the deviation  $\delta M_\nu$  in the neutrino mass matrix from a tree-level, say tribimaximal neutrino mass matrix, does not depend on  $m_0$ . As a result  $\delta M_\nu$ 's are exactly predicted in terms of the experimentally determined quantities such as solar and atmospheric mass squared differences and the mixing angles. On the other hand for  $m_0 \gtrsim 10^{-3}\text{eV}$ ,  $\delta M_\nu$  strongly depends on  $m_0$  and hence can not be determined within the knowledge of oscillation parameters alone. In this limit, we provide an exponential parameterization for  $\delta M_\nu$  for all values of  $m_0$  such that it can factorize the  $m_0$  dependency of  $\delta M_\nu$  from rest of the oscillation parameters. This helps us in finding  $\delta M_\nu$  as a function of the solar and atmospheric mass squared differences and the mixing angles for any values of  $m_0$ . We use this information to build up a model of neutrino masses and mixings in a top-down scenario which can predict large  $\theta_{13}$  perturbatively.

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## I. INTRODUCTION

Compelling evidences of neutrino oscillations observed in solar, atmospheric, and reactor experiments indicate that neutrinos are massive and hence they mix with each other [1]. In the basis where the charged lepton masses are real and diagonal, the mixing matrix is given by [2]:

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{\text{CP}}} \\ -c_{23} s_{12} - s_{23} c_{12} s_{13} e^{i\delta_{\text{CP}}} & c_{23} c_{12} - s_{23} s_{12} s_{13} e^{i\delta_{\text{CP}}} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} c_{12} s_{13} e^{i\delta_{\text{CP}}} & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i\delta_{\text{CP}}} & c_{23} c_{13} \end{pmatrix} P, \quad (1)$$

where  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$  and  $\delta_{\text{CP}}$  is the Dirac CP violating phase which appear in oscillation experiments. The diagonal matrix  $P = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$ , consists of two Majorana phases  $\phi_1$  and  $\phi_2$  which are not relevant in neutrino oscillation experiments [3, 4]. However, they affect lepton number violating amplitudes such as neutrinoless double beta decay. Thus in a flavor basis, where charged leptons are real and diagonal, the neutrino mass matrix is given by:

$$M_\nu = U_{\text{PMNS}}^* \text{diag}(m_1, m_2, m_3) U_{\text{PMNS}}^\dagger, \quad (2)$$

where  $m_1, m_2, m_3$  are the mass eigenvalues.

In the last decade, data from solar and reactor neutrino experiments have provided information on the sign and magnitude of  $\Delta m_{\odot}^2$  and a precise value of  $\theta_{12}$  [5, 6]. The atmospheric parameters  $|\Delta m_{\text{atm}}^2|$  and  $\theta_{23}$  have been measured and their precision will be increased by T2K [7] and NO $\nu$ A [8]. The sign of solar mass splitting  $\Delta m_{\odot}^2$  is precisely known, while the sign of atmospheric mass splitting  $\Delta m_{\text{atm}}^2$  is still unknown. This opens up a possibility of whether neutrino masses follow normal ordering, i.e.,  $m_1 < m_2 < m_3$  or inverted ordering, i.e.,  $m_3 < m_1 < m_2$ . In other words, the lightest mass, either  $m_1$  (normal ordering) or  $m_3$  (inverted ordering) is yet to be determined. Recent measurement from T2K [9], MINOS [10], Double Chooz [11], Daya Bay [12], and RENO [13] confirms a non-zero value of  $\theta_{13}$  at  $5\sigma$  confidence level. This opens up a range of possibilities to measure the sign of the atmospheric mass splitting and the unknown CP phase  $\delta_{\text{CP}}$ .

The absolute mass scale of neutrino is hitherto not known and can only be measured in a tritium beta decay experiment. The KATRIN experiment, which will investigate the kinematics of tritium beta decay, aims to measure the neutrino mass with a sensitivity of 0.2 eV [14]. At present the best upper limit at 95% confidence level on the sum of the neutrino masses comes from the cosmic

microwave background data and is given by [15]:

$$\sum_i m_i < 0.44 \text{eV} . \quad (3)$$

Once the absolute mass scale of the lightest neutrino, either  $m_1$  in normal ordering or  $m_3$  in inverted ordering, is determined one can reconstruct the neutrino mass matrix in the flavor basis using the experimental values of the elements of  $U_{\text{PMNS}}$  matrix. This may unravel exact symmetries in the neutrino mass matrix.

In this paper we develop a perturbative bottom-up approach to unravel the flavor structure of neutrino mass matrix. We set the full neutrino mass matrix:  $M_\nu = (M_\nu)_0 + \delta M_\nu$ , where  $\delta M_\nu$  is the perturbation around the tree-level mass matrix  $(M_\nu)_0$ , which is determined using some of the well known mixing scenarios such as tribimaximal (TBM) mixing [16], bimaximal (BM) mixing [17] and/or democratic (DC) mixing [18]. Among all these, the TBM is closer to the experimentally observed mixing pattern and hence mostly studied. All these mixing scenarios, however, predict  $\theta_{13}$  to be zero and hence ruled out by the recent measurement on the reactor neutrino angle. However, a perturbative approach to realize a large value of  $\theta_{13}$  is still a viable option. So, we assume that the non-zero value of  $\theta_{13}$  is generated perturbatively. Using the  $3\sigma$  range of values of the elements of  $U_{\text{PMNS}}$  matrix we determine  $\delta M_\nu$  as a function of the lightest neutrino mass, say  $m_0$ , where  $m_0 = m_1$  in the normal ordering and  $m_0 = m_3$  in the inverted ordering. In this way we find that for  $m_0 < 10^{-3}$  eV, all the  $\delta M_\nu$  are independent of  $m_0$  and hence the lightest neutrino mass does not play any role in the perturbative determination of neutrino mass matrix. Such models, for example, can be generated in two right-handed neutrino extensions of the standard model (SM). However, for  $m_0 \gtrsim 10^{-3}$  eV the lightest neutrino mass plays an important role in the perturbative determination of neutrino mass matrix. We factorize the  $m_0$  dependency of  $\delta M_\nu$  using an exponential parameterization:  $\delta M_\nu(m_0) \propto \delta M_\nu|_{m_0=0} \text{Exp}[-m_0/0.1 \text{eV}]$ . This helps us in finding the required perturbations of  $\delta M_\nu$  as a function of experimentally determined quantities such as solar and atmospheric mass squared differences and the mixing angles. Using this results from the bottom-up approach we provide an example in the top-down scenario which predict large  $\theta_{13}$  perturbatively.

The paper is organized as follows. In section II A, we start with a brief description of the bottom-up approach that is developed in this paper to find the desired perturbation of the mass matrix in the flavor basis. Then, in section II B, we use the knowledge of our current experimental results to

find the deviation in the mass matrix in the flavor basis using the tree level mass matrix that is generated using the TBM mixing ansatz. In section III, we suggest a typical top-down approach in which the perturbation to the TBM mixing is obtained. We then fit the model parameters using the data from the bottom-up approach of section II B and conclude in section IV.

## II. PHENOMENOLOGY

### A. Bottom up approach

The recent global fit to the neutrino oscillation data has ruled out the possibility of a zero reactor angle at  $10.2\sigma$  confidence level [19, 20]. Evidence of non zero  $\theta_{13}$  at  $3\sigma$  level was first established by data from T2K [7], MINOS [10] and Double Chooz [11]. More recently, Daya Bay [12] and RENO [13] experiments confirm large  $\theta_{13}$  at more than  $5\sigma$  confidence level (CL) from the reactor  $\bar{\nu}_e \rightarrow \bar{\nu}_e$  oscillations. The current best fit values and the  $3\sigma$  allowed values of all the mixing parameters are summarized in Table. I. The values written within brackets are for inverted ordering

Oscillation parameters	Best fit value	$3\sigma$ range
$\Delta m_{\odot}^2 \times (10^5 \text{ eV}^2)$	7.62	[7.12 – 8.20]
$ \Delta m_{\text{atm}}^2  \times (10^3 \text{ eV}^2)$	2.55 (2.43)	[2.31 – 2.74] ([2.21 – 2.64])
$\sin^2 \theta_{12}$	0.320	[0.27 – 0.37]
$\sin^2 \theta_{23}$	0.613 (0.600)	[0.36 – 0.68] ([0.37 – 0.67])
$\sin^2 \theta_{13}$	0.0246 (0.0250)	[0.017 – 0.033]

TABLE I: Current status of oscillation parameters taken from Ref. [19].

of the neutrino mass spectrum. At this juncture we note that the only unknown quantity in the neutrino mass matrix is the lightest neutrino mass apart from the CP violating phases (one Dirac and two Majorana phases)

The neutrino mixing matrix derived from the current best fit parameters is

$$U_{\text{EXPT}}^c = \begin{pmatrix} 0.81908 & 0.54773 & 0.17058 \\ -0.49795 & 0.53116 & 0.68551 \\ 0.28487 & -0.64642 & 0.70780 \end{pmatrix}, \quad (4)$$



and the corresponding  $3\sigma$  allowed values of the mixing matrix is

$$U_{\text{EXPT}} = \begin{pmatrix} 0.78 - 0.85 & 0.51 - 0.60 & 0.13 - 0.18 \\ -(0.39 - 0.57) & 0.36 - 0.64 & 0.59 - 0.82 \\ 0.19 - 0.44 & -(0.54 - 0.76) & 0.56 - 0.79 \end{pmatrix}, \quad (5)$$

where we have assumed  $\delta_{CP}$  to be zero for simplicity. The corresponding neutrino mass matrix in the flavor basis can be written as

$$(M_\nu)_{\text{EXPT}} = U_{\text{EXPT}} \text{diag}(m_1, m_2, m_3) U_{\text{EXPT}}^T \quad (6)$$

where  $(M_\nu)_{\text{EXPT}}$  is a real symmetric matrix. Therefore, it is described by three masses:  $m_1, m_2, m_3$  and three mixing angles:  $\theta_{12}, \theta_{23}, \theta_{13}$ . The mass eigenvalues  $m_1, m_2, m_3$  in the basis where the charged leptons are real and diagonal can be expressed as:  $m_2 = \sqrt{\Delta m_\odot^2 + m_1^2}$  and  $m_3 = \sqrt{\Delta m_{\text{atm}}^2 + m_1^2}$  for normal hierarchy and  $m_1 = \sqrt{m_3^2 + \Delta m_{\text{atm}}^2}$  and  $m_2 = \sqrt{m_3^2 + \Delta m_{\text{atm}}^2 + \Delta m_\odot^2}$  for inverted hierarchy.

We define the perturbed neutrino mass matrix  $\delta M_\nu$  as

$$\delta M_\nu = (M_\nu)_{\text{EXPT}} - (M_\nu)_0, \quad (7)$$

where  $(M_\nu)_{\text{EXPT}}$  is the neutrino mass matrix given by Eq. 6 and  $(M_\nu)_0$  is the neutrino mass matrix corresponding to a mixing matrix  $U_0$ , that is obtained in a specific model such as TBM mixing, BM mixing and/or DC mixing *etc.* This  $\delta M_\nu$  is a symmetric matrix whose elements denote the amount of perturbation that is needed to add to the  $(M_\nu)_0$  so that it is consistent within the  $3\sigma$  of the experimental results. In order to gauge the size of the  $\delta M_\nu$  matrix, we do a random scan of all the parameters:  $\theta_{13}, \theta_{23}, \theta_{12}, \Delta m_\odot^2$  and  $|\Delta m_{\text{atm}}^2|$  within their  $3\sigma$  allowed range of values. Note that  $(M_\nu)_0$  depends only on the value of the lightest neutrino mass  $m_0$  as the mixing angles are expected to be fixed via certain symmetries as done in TBM, BM and/or DC mixing scenarios. Hence, to see the effect of  $m_0$  on the elements of  $\delta M_\nu$  we vary  $m_0$  in the range  $[10^{-9}, 10]$  eV. We then define the predicted neutrino mass matrix  $(M_\nu)_M$  of any model as:

$$(M_\nu)_M = (M_\nu)_0 + \delta M_\nu. \quad (8)$$

Note that the matrix elements of  $(M_\nu)_M$  and  $\delta M_\nu$  are function of the oscillation parameters  $\theta_{12}, \theta_{23}$ , and  $\theta_{13}$  as well as of the lightest neutrino mass  $m_0$ . We then perform a naive statistical analysis.

We compare the elements of the  $(M_\nu)_M$  matrix to the experimental data of Eq. 6 by a  $\chi^2$  function which is defined by:

$$\chi^2 = \left[ (M_\nu^c)_{\text{EXPT}} - (M_\nu)_M \right]^2 / m_0 \sigma^2 = \left[ \delta M_\nu^c - \delta M_\nu \right]^2 / m_0 \sigma^2, \quad (9)$$

where  $\sigma = \sqrt{(M_\nu^c)_{\text{EXPT}}}$  and  $\delta M_\nu^c = (M_\nu^c)_{\text{EXPT}} - (M_\nu)_0$ . Note that  $(M_\nu^c)_{\text{EXPT}}$  is the experimental mass matrix corresponding to the best fit values of the oscillation parameters and is diagonalized by the mixing matrix of Eq. 4. It is worth noting that as  $\delta M_\nu \rightarrow \delta M_\nu^c$ , we get the minimum  $\chi^2$  and hence  $\delta M_\nu^c$  is the required amount of perturbation that should be added to  $(M_\nu)_0$ . We will discuss the properties of the  $\delta M_\nu$  matrix using the TBM mixing ansatz. Although we focus mainly on the TBM mixing, some of the features presented here are applicable to other tree-level mass matrix.

## B. Perturbation in Tribimaximal mixing Scenario

We proceed to discuss the tribimaximal (TBM) mixing scenario. For the TBM mixing, we have  $s_{12} = 1/\sqrt{3}$ ,  $s_{23} = 1/\sqrt{2}$ , and  $s_{13} = 0$ . We know that apart from the recently measured value of  $\theta_{13}$ , the solar and atmospheric mixing angles are in a very good agreement with the TBM ansatz. Modifications to this TBM scenario have been studied by many authors [21].

We begin by writing the TBM mixing matrix in the standard parameterization of Eq. 1, that is

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ \sqrt{1/6} & -\sqrt{1/3} & \sqrt{1/2} \end{pmatrix} = \begin{pmatrix} 0.81650 & 0.57735 & 0 \\ -0.40825 & 0.57735 & 0.70711 \\ 0.40825 & -0.57735 & 0.70711 \end{pmatrix}. \quad (10)$$

The corresponding neutrino mass matrix in the flavor basis and the perturbed matrix  $\delta M_\nu$  are

$$(M_\nu)_{\text{TBM}} = U_{\text{TBM}} \text{diag}(m_1, m_2, m_3) U_{\text{TBM}}^T, \quad \delta M_\nu = (M_\nu)_{\text{EXPT}} - (M_\nu)_{\text{TBM}}. \quad (11)$$

We wish to determine the elements of this  $\delta M_\nu$  matrix in a bottom-up approach. We follow the procedure given in section II A and perform a random scan over all the oscillation parameters within their  $3\sigma$  range of values. In Fig. 1, we show  $\delta M_\nu(i, j)$  versus  $\chi^2$  for  $m_0 = 10^{-9}$  eV. The minimum of each curve corresponds to the value of  $\delta M_\nu^c(i, j)$ . A similar plot is shown in Fig. 2 for inverted hierarchy as well. We know that the neutrino mass matrix in the flavor basis does depend on the value of the lightest eigenvalue  $m_0$ . In order to see the effect of  $m_0$  on the value of  $\delta M_\nu^c(i, j)$  qualitatively, we vary  $m_0$  in the range  $(10^{-10}, 10)$  eV while keeping other parameters such as  $\theta_{12}$ ,  $\theta_{13}$ ,

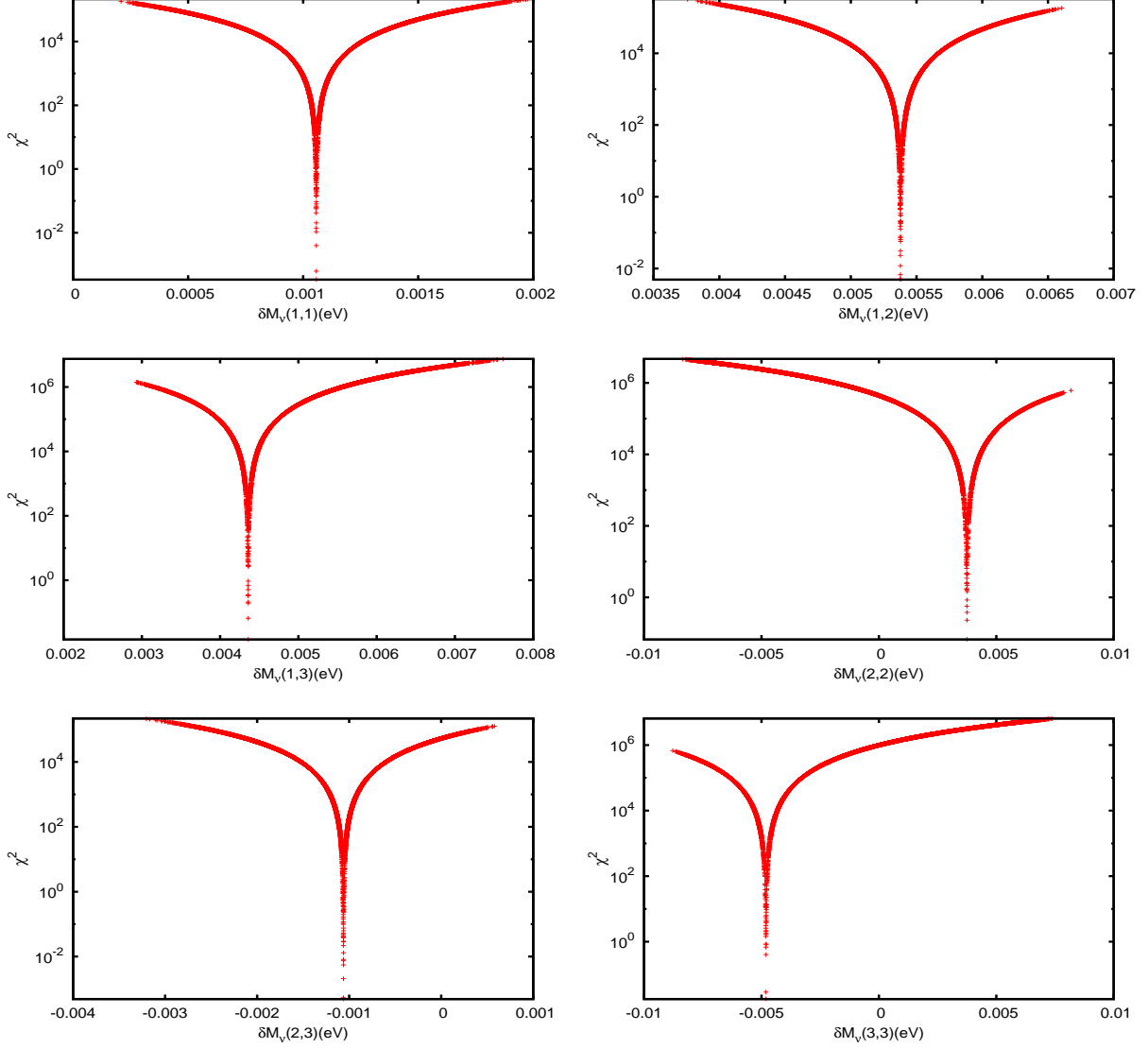


FIG. 1:  $\chi^2$  of  $\delta M_\nu(i, j)$  for normal ordering of neutrino masses with TBM mixing in tree-level.

$\theta_{23}$ ,  $|\Delta m_{\text{atm}}^2|$ , and  $\Delta m_\odot^2$  fixed at their best fit values. In Fig. 3, we plot  $\delta M_\nu^c(i, j)$  versus  $m_0$  and it is clear that for  $m_0 < 10^{-3}$  eV, the value of  $\delta M_\nu^c(i, j)$  remains constant. In other words,  $\delta M_\nu(i, j)$  are independent of the value of  $m_0$ . However, for  $m_0 \gtrsim 10^{-3}$  eV, the value of  $\delta M_\nu(i, j)$  keeps changing. In the degenerate limit when  $m_1 \sim m_2 \sim m_3$ ,  $\delta M_\nu(i, j)$  become zero. This particular feature is common to both normal and inverted hierarchies as can be seen from Fig. 3. To appreciate the plots in Fig. 3, we write down  $\delta M_\nu(i, j)$  as a function of neutrino masses and mixing angles:

$$\delta M_\nu(1, 1) = ((c_{12}c_{13})^2 - 2/3)m_1 + ((s_{12}c_{13})^2 - 1/3)m_2 + s_{13}^2 m_3$$

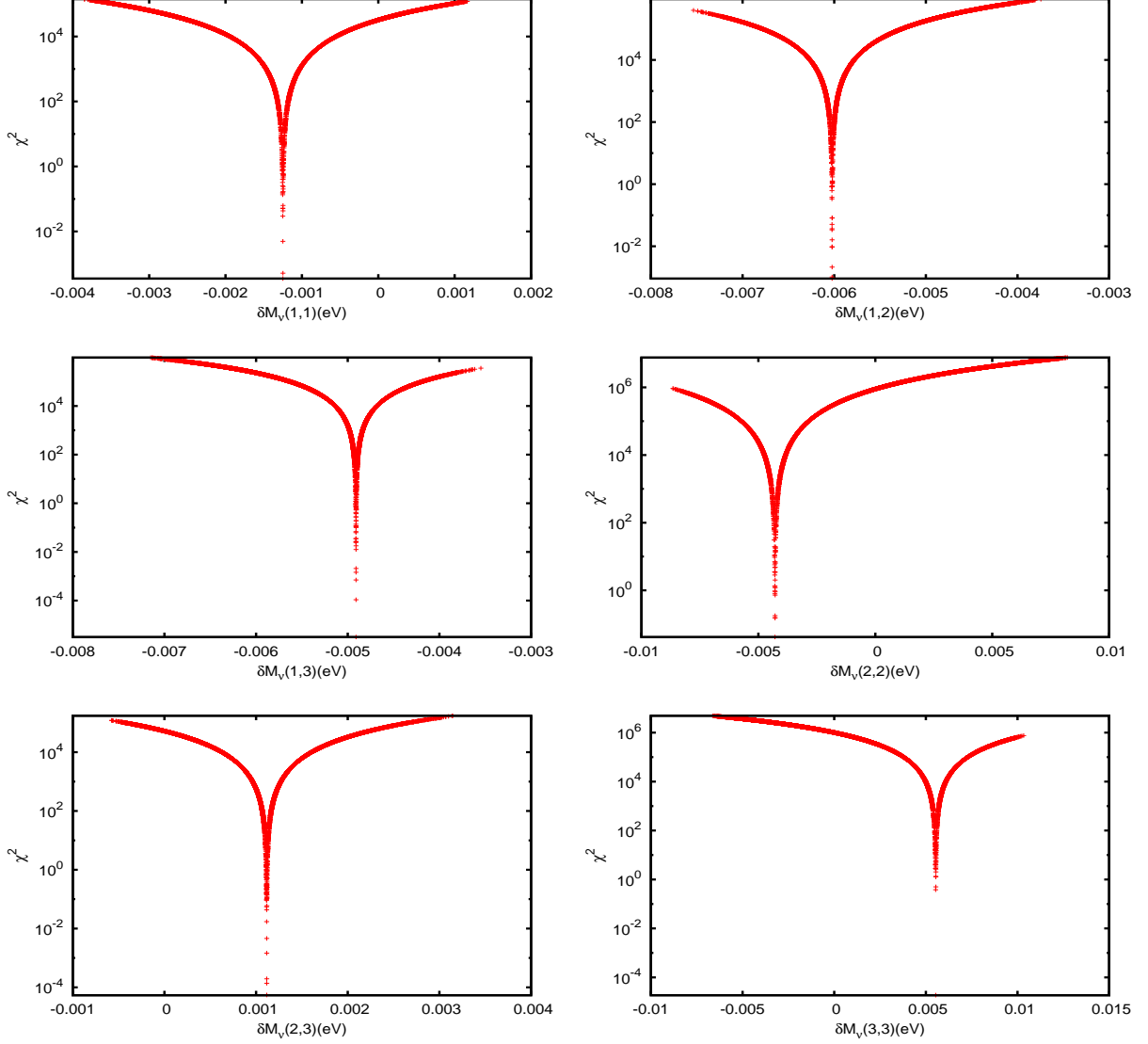


FIG. 2:  $\chi^2$  of  $\delta M_\nu(i, j)$  for inverted ordering of neutrino masses with TBM mixing in tree-level.

$$\begin{aligned}
\delta M_\nu(1, 2) &= (c_{12}c_{13}(-s_{12}c_{23} - c_{12}s_{23}s_{13}) + 1/3)m_1 \\
&\quad + (s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{23}s_{13}) - 1/3)m_2 + s_{13}c_{13}s_{23}m_3 \\
\delta M_\nu(1, 3) &= (c_{12}c_{13}(s_{12}s_{23} - c_{12}c_{23}s_{13}) - 1/3)m_1 \\
&\quad + (s_{12}c_{13}(-c_{12}s_{23} - s_{12}c_{23}s_{13}) + 1/3)m_2 + s_{13}c_{13}c_{23}m_3 \\
\delta M_\nu(2, 2) &= ((-s_{12}c_{23} - c_{12}s_{23}s_{13})^2 - 1/6)m_1 \\
&\quad + ((c_{12}c_{23} - s_{12}s_{23}s_{13})^2 - 1/3)m_2 + ((c_{13}s_{23})^2 - 1/2)m_3
\end{aligned}$$

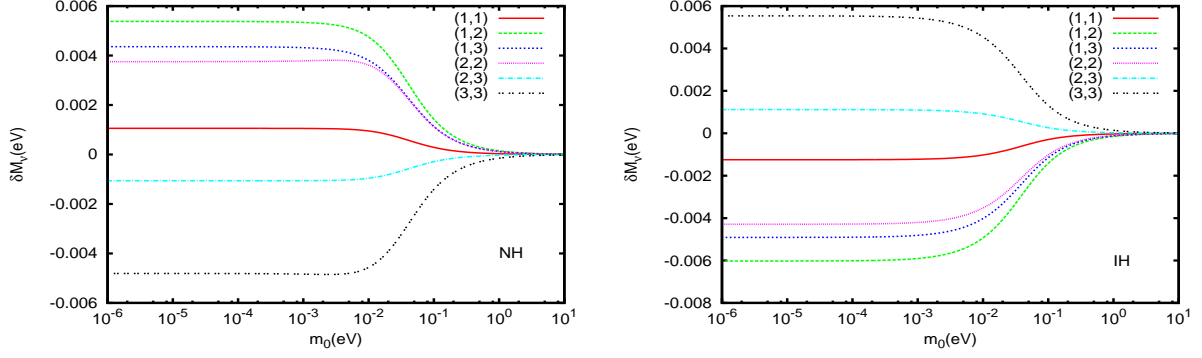


FIG. 3: Variation of  $\delta M_\nu$  with respect to the lightest neutrino mass for NH (left) and IH (right) of neutrino masses with TBM mixing.

$$\begin{aligned}
\delta M_\nu(2,3) &= ((-s_{12}c_{23} - c_{12}s_{23}s_{13})(s_{12}s_{23} - c_{12}c_{23}s_{13}) + 1/6)m_1 \\
&\quad + ((c_{12}c_{23} - s_{12}s_{23}s_{13})(-c_{12}s_{23} - s_{12}c_{23}s_{13}) + 1/3)m_2 + (s_{23}c_{13}c_{23}c_{13} - 1/2)m_3 \\
\delta M_\nu(3,3) &= ((s_{12}s_{23} - c_{12}c_{23}s_{13})^2 - 1/6)m_1 \\
&\quad + ((-c_{12}s_{23} - s_{12}c_{23}s_{13})^2 - 1/3)m_2 + ((c_{13}c_{23})^2 - 1/2)m_3.
\end{aligned} \tag{12}$$

Note that the higher eigenvalues  $m_2$  and  $m_3(m_1)$  can be re-expressed as a function of lightest neutrino mass  $m_0 \equiv m_1(m_3)$ . Therefore, it is obvious that all the elements:  $\delta M_\nu(i, j)$  are function of the only unknown quantity  $m_0$ . From Fig. 3 we see that  $\delta M_\nu(1, 1)$ ,  $\delta M_\nu(1, 2)$ ,  $\delta M_\nu(1, 3)$ ,  $\delta M_\nu(2, 2)$  start with a positive value, while  $\delta M_\nu(2, 3)$  and  $\delta M_\nu(3, 3)$  start with a negative value. An exactly opposite spectrum is observed in case of inverted hierarchy as expected. In either case we observe that for  $m_0 < 10^{-3}$  eV,  $\delta M_\nu(i, j)$  are independent of  $m_0$ . Therefore, in this limit it is reasonable to set  $m_0 \approx 0$ . In the opposite limit when  $m_0 \gg \sqrt{|\Delta m_{\text{atm}}^2|} = 4.9 \times 10^{-2}$  eV, we get  $m_3 \approx m_2 \approx m_1$ . In this limit  $M_\nu(i, j) \approx m_0 (I)$ , where  $I$  is the identity matrix. Hence all the  $\delta M_\nu(i, j)$  are zero. For  $m_0 \approx 10^{-3}$  eV, it is comparable to solar and atmospheric values. Therefore from Fig. 3 we see an appreciable effect of  $m_0$  on various  $\delta M_\nu(i, j)$ . We have factored out this dependency of all  $\delta M_\nu(i, j)$  on  $m_0$  using an exponential parameterization:

$$\begin{aligned}
\left| \delta M_\nu(m_0, \sqrt{\Delta m_\odot^2}, \sqrt{|\Delta m_{\text{atm}}^2|}, \theta_{12}, \theta_{13}, \theta_{23}) \right| &= \left| \delta M_\nu(m_0 = 0, \sqrt{\Delta m_\odot^2}, \sqrt{|\Delta m_{\text{atm}}^2|}, \theta_{12}, \theta_{13}, \theta_{23}) \right| \\
&\quad \times \text{Exp}(-m_0/a).
\end{aligned} \tag{13}$$

where  $a$  is determined from a fit to the recent experimental data and found to be  $a \approx 0.1$  eV. As a

result we could do all our analysis for  $m_0 \leq 10^{-3} \text{ eV}$  which is equivalent to setting  $m_0 = 0$ . Then we generalize it to any value of  $m_0$  using Eq. 13.

We first compare the perturbed matrix elements  $\delta M_\nu^c(i, j)$  with the elements of tree level mass matrix  $(M_\nu)_{\text{TBM}}(i, j)$ . In case of normal hierarchy, the experimental mass matrix, the tree-level neutrino mass matrix, and its deviations from the central values are given by:

$$\frac{(M_\nu^c)_{\text{EXPT}}}{10^{-2} \text{ eV}} = \begin{pmatrix} 0.397 & 0.829 & 0.145 \\ 0.829 & 3.191 & 2.128 \\ 0.145 & 2.128 & 2.335 \end{pmatrix}, \quad \frac{(M_\nu)_{\text{TBM}}}{10^{-2} \text{ eV}} = \begin{pmatrix} 0.291 & 0.291 & -0.291 \\ 0.291 & 2.816 & 2.234 \\ -0.291 & 2.234 & 2.816 \end{pmatrix}, \quad (14)$$

and

$$\frac{\delta M_\nu^c}{10^{-2} \text{ eV}} = \begin{pmatrix} 0.106 & 0.538 & 0.436 \\ 0.538 & 0.375 & -0.106 \\ 0.436 & -0.106 & -0.481 \end{pmatrix}. \quad (15)$$

Similarly, for inverted hierarchy, the mass matrix  $(M_\nu^c)_{\text{EXPT}}$  and  $(M_\nu)_{\text{TBM}}$  are,

$$\frac{(M_\nu^c)_{\text{EXPT}}}{10^{-2} \text{ eV}} = \begin{pmatrix} 4.830 & -0.577 & -0.517 \\ -0.577 & 2.061 & -2.379 \\ -0.517 & -2.379 & 3.044 \end{pmatrix}, \quad \frac{(M_\nu)_{\text{TBM}}}{10^{-2} \text{ eV}} = \begin{pmatrix} 4.956 & 0.026 & -0.026 \\ 0.026 & 2.490 & -2.490 \\ -0.026 & -2.490 & 2.490 \end{pmatrix} \quad (16)$$

and the corresponding perturbed matrix is

$$\frac{\delta M_\nu^c}{10^{-2} \text{ eV}} = \begin{pmatrix} -0.125 & -0.602 & -0.491 \\ -0.602 & -0.429 & 0.112 \\ -0.491 & 0.112 & 0.554 \end{pmatrix}. \quad (17)$$

We introduce a new parameter  $\epsilon$  as:

$$\epsilon^c(i, j) = \frac{\delta M_\nu^c(i, j)}{(M_\nu)_{\text{TBM}}(i, j)} \quad (18)$$

such that it will give a measure of required perturbation with respect to the corresponding tree-level value. Thus for normal and inverted hierarchies of neutrino mass spectrum, we get:

$$(\epsilon^c)_{\text{NH}} = \begin{pmatrix} 0.364 & 1.848 & -1.848 \\ 1.848 & 0.133 & -0.047 \\ -1.848 & -0.047 & 0.171 \end{pmatrix} \quad (\epsilon^c)_{\text{IH}} = \begin{pmatrix} 0.02 & 23.154 & 18.885 \\ 23.154 & 0.172 & 0.045 \\ 18.885 & 0.045 & 0.222 \end{pmatrix}. \quad (19)$$

From Eq. (19), it is clear that (1, 2), and (1, 3) elements of the TBM mass matrix needs to be modified largely to be consistent with the experiment. The perturbations of rest of the elements are small in comparison to their tree-level masses. In case of NH the modification is mild, while it is significant in case of IH case.

We now try to explore the  $\theta_{13}$ ,  $\theta_{23}$ , and  $\theta_{12}$  dependency of all the elements of the neutrino mass matrix in the flavor basis where the charged lepton is assumed to be diagonal. In order to see the effect of a particular oscillation parameter on the value of  $\delta M_\nu$ , we vary that parameter in the  $3\sigma$  allowed range and do a marginalization over the other oscillation parameters. From Eq. (12) we see that in either case of NH and IH the dependency of  $\delta M_\nu(1, 2)$  and  $\delta M_\nu(1, 3)$  on  $\theta_{12}$  is almost negligible because of the cancellation in the second term of each expression. However, both  $\delta M_\nu(1, 2)$  and  $\delta M_\nu(1, 3)$  do depend on  $\theta_{23}$  and  $\theta_{13}$  in either case of NH and IH as can be seen from Fig. 4. The dependency of  $\delta M_\nu(1, 2)$  and  $\delta M_\nu(1, 3)$  elements on  $\theta_{13}$  and  $\theta_{23}$  can be understood from the following analytical approximation. In case of NH,  $m_1 \rightarrow 0$ . Hence we have  $m_3 = \sqrt{\Delta m_{\text{atm}}^2}$  and  $m_2 = \sqrt{\Delta m_\odot^2}$ . As a result the second term of  $\delta M_\nu(1, 2)$  and  $\delta M_\nu(1, 3)$  is proportional to  $\sqrt{\Delta m_\odot^2} \ll \sqrt{\Delta m_{\text{atm}}^2}$ . Moreover, there is a cancellation in the second term for which the second term is suppressed in comparison to the third term. Therefore, we get

$$\begin{aligned}\delta M_\nu(1, 2) &\approx \sqrt{\Delta m_{\text{atm}}^2} \sin \theta_{13} \cos \theta_{13} \sin \theta_{23} \\ \delta M_\nu(1, 3) &\approx \sqrt{\Delta m_{\text{atm}}^2} \sin \theta_{13} \cos \theta_{13} \cos \theta_{23} .\end{aligned}\quad (20)$$

On the other hand in case of IH,  $m_3 \rightarrow 0$ . Hence we have  $m_2 \approx m_1 = \sqrt{\Delta m_{\text{atm}}^2}$ . As a result we get

$$\begin{aligned}|\delta M_\nu(1, 2)| &\approx \sqrt{\Delta m_{\text{atm}}^2} \sin \theta_{13} \cos \theta_{13} \sin \theta_{23} \\ |\delta M_\nu(1, 3)| &\approx \sqrt{\Delta m_{\text{atm}}^2} \sin \theta_{13} \cos \theta_{13} \cos \theta_{23} .\end{aligned}\quad (21)$$

Thus in either cases the dependency of  $\delta M_\nu(1, 2)$  and  $\delta M_\nu(1, 3)$  elements on  $\theta_{13}$  and  $\theta_{23}$  are very similar. This can be checked from Fig. 4. It is evident from Fig. 4 that  $\delta M_\nu(2, 2)$ ,  $\delta M_\nu(2, 3)$ , and  $\delta M_\nu(3, 3)$  elements does not depend on  $\theta_{13}$  and  $\theta_{12}$  at all. It only depends on the value of  $\theta_{23}$ . To understand this feature quantitatively, we write down the matrix elements  $\delta M_\nu(2, 2)$ ,  $\delta M_\nu(2, 3)$ , and  $\delta M_\nu(3, 3)$  for  $m_0 = 0$ . For normal ordering of the neutrino mass spectrum, we have

$$\begin{aligned}\delta M_\nu(2, 2) &\approx (\cos^2 \theta_{13} \sin^2 \theta_{23} - \frac{1}{2}) \sqrt{\Delta m_{\text{atm}}^2} , \\ \delta M_\nu(2, 3) &\approx (\sin \theta_{23} \cos \theta_{23} \cos^2 \theta_{13} - \frac{1}{2}) \sqrt{\Delta m_{\text{atm}}^2} , \\ \delta M_\nu(3, 3) &\approx (\cos^2 \theta_{13} \cos^2 \theta_{23} - \frac{1}{2}) \sqrt{\Delta m_{\text{atm}}^2}\end{aligned}\quad (22)$$

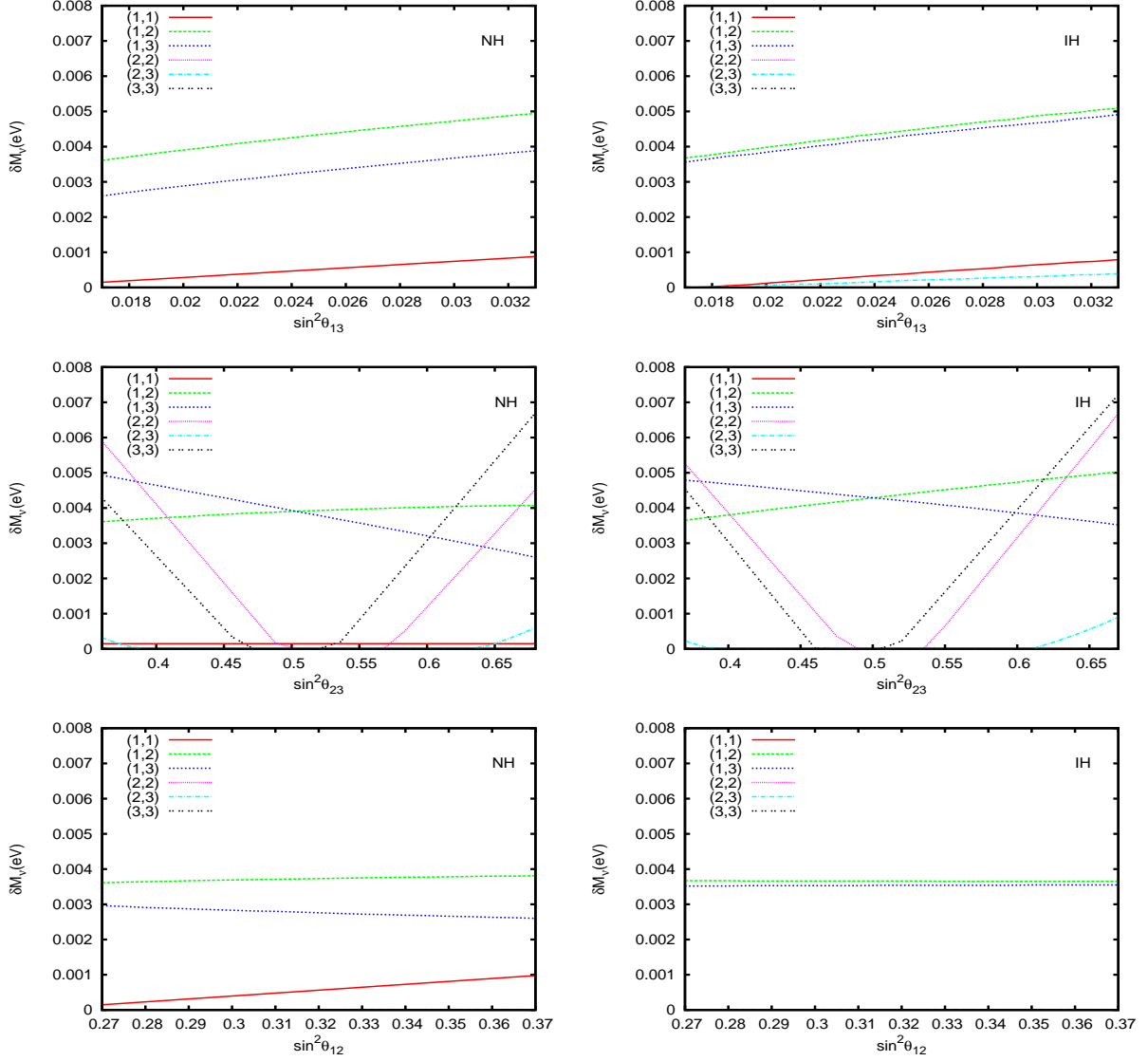


FIG. 4:  $\delta M_\nu(i, j)$  as function of mixing angles for  $m_0 = 0$  in case of NH (left) and IH (right) of neutrino mass matrix.

Clearly,  $\delta M_\nu(2, 2)$ ,  $\delta M_\nu(2, 3)$ , and  $\delta M_\nu(3, 3)$  does not depend on  $\theta_{12}$  at all. There is  $\theta_{13}$  dependency and is of similar order if  $\theta_{23}$  is near to the TBM value. However, once the value of  $\theta_{23}$  deviates away from the  $\theta_{23}$  (TBM) value, all these matrix elements mainly depend on the value of  $\theta_{23}$  and the value of  $\delta M_\nu(i, j)$  increases as we move away from the TBM value. In case of IH,  $m_3 \rightarrow 0$  and a similar pattern for all the  $\delta M_\nu(i, j)$  is expected. This can be easily read from the right panel of Fig. 4.



### III. TOP-DOWN APPROACH

Now that we know the perturbed matrix from the bottom-up approach, we wish to construct models of neutrino masses and mixings what we call top-down approach. Our main objective here is that we will use the predictions of bottom-up approach as guide lines for building neutrino mass matrix whose tree-level mixing is governed by a TBM ansatz. We modify the TBM mixing matrix using some perturbation so that the resulting values are consistent with the results that are obtained using the bottom up approach of section. II B. We begin by proposing a model where the neutrino mixing is described by  $U_M = U_{\text{TBM}} V$ , where  $V$  is the perturbed mixing matrix around the TBM value and in general is given by:

$$V = V(1, 2) V(1, 3) V(2, 3), \quad (23)$$

with  $V(i, j)$  being given by an orthogonal rotation matrix in the  $(i, j)$  plane of neutrino mass matrix. Let us write down  $V(i, j)$  explicitly such that the perturbed mixing matrix is

$$\begin{aligned} V = V(1, 2) V(1, 3) V(2, 3) &= \begin{pmatrix} 1 & \beta & 0 \\ -\beta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 1 & 0 \\ -\alpha & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \gamma \\ 0 & -\gamma & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & \beta - \alpha\gamma & \alpha + \beta\gamma \\ -\beta & 1 & \gamma - \beta\alpha \\ -\alpha & -\gamma & 1 \end{pmatrix}, \end{aligned} \quad (24)$$

where the determinant of  $V(i, j)$  matrix is assumed to be unity. Since  $U_M$  is the diagonalising matrix of the proposed neutrino mass matrix, we get

$$\begin{aligned} M_\nu^D &= U_M^T (M_\nu)_M U_M = U_M^T ((M_\nu)_{\text{TBM}} + \delta M_\nu) U_M \\ &= V^T U_{\text{TBM}}^T ((M_\nu)_{\text{TBM}} + \delta M_\nu) U_{\text{TBM}} V, \end{aligned} \quad (25)$$

where,  $(M_\nu)_M$  is the mass matrix in a flavor basis where charged leptons are real and diagonal. We now define a matrix  $X$  such that

$$X = V M_\nu^D V^T = U_{\text{TBM}}^T ((M_\nu)_{\text{TBM}} + \delta M_\nu) U_{\text{TBM}}, \quad (26)$$

which leads to the following equations in terms of  $\alpha$ ,  $\beta$ , and  $\gamma$  in the  $m_1 \rightarrow 0$  limit.

$$X_{11} = m_2 \beta^2 + m_3 \alpha^2,$$

$$\begin{aligned}
X_{12} &= m_2 \beta + (m_3 - m_2) \alpha \gamma, \\
X_{13} &= m_3 \alpha + (m_3 - m_2) \beta \gamma, \\
X_{22} &= m_2 + m_3 \gamma^2, \\
X_{23} &= -m_3 \alpha \beta + (m_3 - m_2) \gamma, \\
X_{33} &= m_3 + m_2 \gamma^2,
\end{aligned} \tag{27}$$

where  $m_2 = \sqrt{\Delta m_{\odot}^2}$  and  $m_3 = \sqrt{\Delta m_{\text{atm}}^2}$ , respectively. Here we have ignored higher order terms in  $\alpha$ ,  $\beta$ , and  $\gamma$ . In Table. II, we report all the elements of the matrix  $X$  obtained using the bottom up approach for the TBM mixing scenarios. We need to find the range of  $\alpha$ ,  $\beta$ , and  $\gamma$  so that  $X(i, j)$

TBM	X(1,1)	X(1,2)	X(1,3)	X(2,2)	X(2,3)	X(3,3)
Central values	0.0203	0.0487	0.315	1.01	0.747	4.89
$3\sigma$ range	$[0.87 \times 10^{-4}, 0.269]$	$[-0.0937, 0.118]$	$[0.0141, 1.15]$	$[0.839, 1.17]$	$[-0.153, 1.05]$	$[4.57, 5.14]$

TABLE II: Values of  $X(i, j) \times 10^2$  for TBM mixing from the bottom up approach.

values computed from Eq. 27 are consistent with the values reported in Table. II. We get the range of  $\alpha$ ,  $\beta$ , and  $\gamma$  by fitting  $X(1, 2)$ ,  $X(1, 3)$  and  $X(2, 2)$  in Eq. 27 with the corresponding  $X(1, 2)$ ,  $X(1, 3)$ , and  $X(2, 2)$  ranges from Table. II. Once the range in  $\alpha$ ,  $\beta$ , and  $\gamma$  are known, we then plug those in Eq. 27 to get the range in  $X(1, 1)$ ,  $X(2, 3)$ , and  $X(3, 3)$  and are shown in Fig. 5. In order to realize the dependence of Eq. 27 on number of parameters, we set  $\gamma$  equal to zero and compare it with  $\gamma \neq 0$  case. The results are shown in Fig. 6. It is clear that once we set  $\gamma = 0$ , we do not get a very good fit for  $X(2, 2)$  and  $X(2, 3)$ . We did the same exercise by setting  $\beta = 0$ . We fix the range of  $\alpha$  and  $\gamma$  from  $X(1, 3)$  and  $X(2, 2)$ . Once the range of  $\alpha$  and  $\gamma$  are fixed, we get the range of  $X(1, 1)$ ,  $X(1, 2)$ ,  $X(2, 3)$ , and  $X(3, 3)$  using Eq. 27 and is shown in Fig. 7. Similarly, once we set  $\alpha = 0$  and fix the range of  $\beta$  and  $\gamma$  from  $X(1, 2)$  and  $X(2, 2)$ , we get the range of  $X(1, 1)$ ,  $X(1, 3)$ ,  $X(2, 3)$ , and  $X(3, 3)$  using Eq. 27. The resulting ranges are shown in Fig. 8. It is clear from Fig. 5 that, we get a very good fit with the experimental data set if all the 3 model parameters ( $\alpha, \beta, \gamma$ ) are non zero. However, setting  $\beta$  and  $\gamma$  to zero also gives a quite good fit to the experimental data even though there are some discrepancies between the data and the theory for some matrix elements. But it is evident from Fig. 8 that, non zero values of  $\beta$  and  $\gamma$  alone can not reproduce the experimental results. Thus, to have a reasonably good fit we at least need to have a non zero  $\alpha$ , *i.e.*,

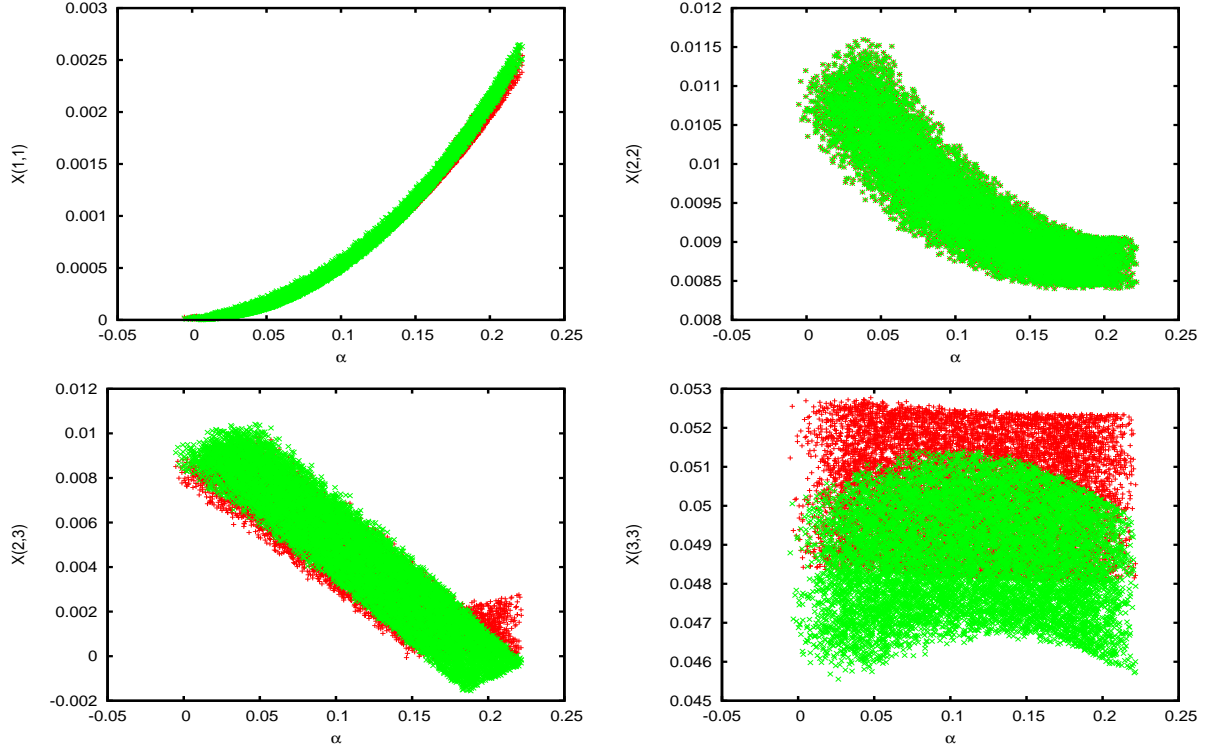


FIG. 5: Range of  $X(i, j)$  as a function of  $\alpha$  for  $\alpha, \beta, \gamma \neq 0$ . The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

the perturbation in the  $(1, 3)$  plane. Again, since we can set both  $\beta$  and  $\gamma$  to be zero, the order in which we do the rotations to construct the perturbed matrix  $V$  is not very important. We further note that if  $V$  is product of atleast two matrices (that is  $V$  is parameterised by atleast two variables) then the parameters in  $V$ -matrix can not be considered as true-perturbation around their tree-level values. Therefore, the new parameters  $\alpha, \beta, \gamma, \dots$  are arbitrary. However, if  $V$  is parameterised by a single variable, say  $\alpha$ , then it can be considered as a true-perturbation around its tree-level value. Such a case can be obtained using an  $A_4$  symmetry in a type-II seesaw framework [16].

#### IV. CONCLUSION

The large value of  $\theta_{13}$  predicted by Daya Bay and RENO put a stringent constraint on theoretical model buildings for neutrino mixing. All the well known mixing ansatz such as TBM mixing, BM mixing, and DC mixing predicts  $\theta_{13}$  to be zero and hence not consistent with experiment. However,

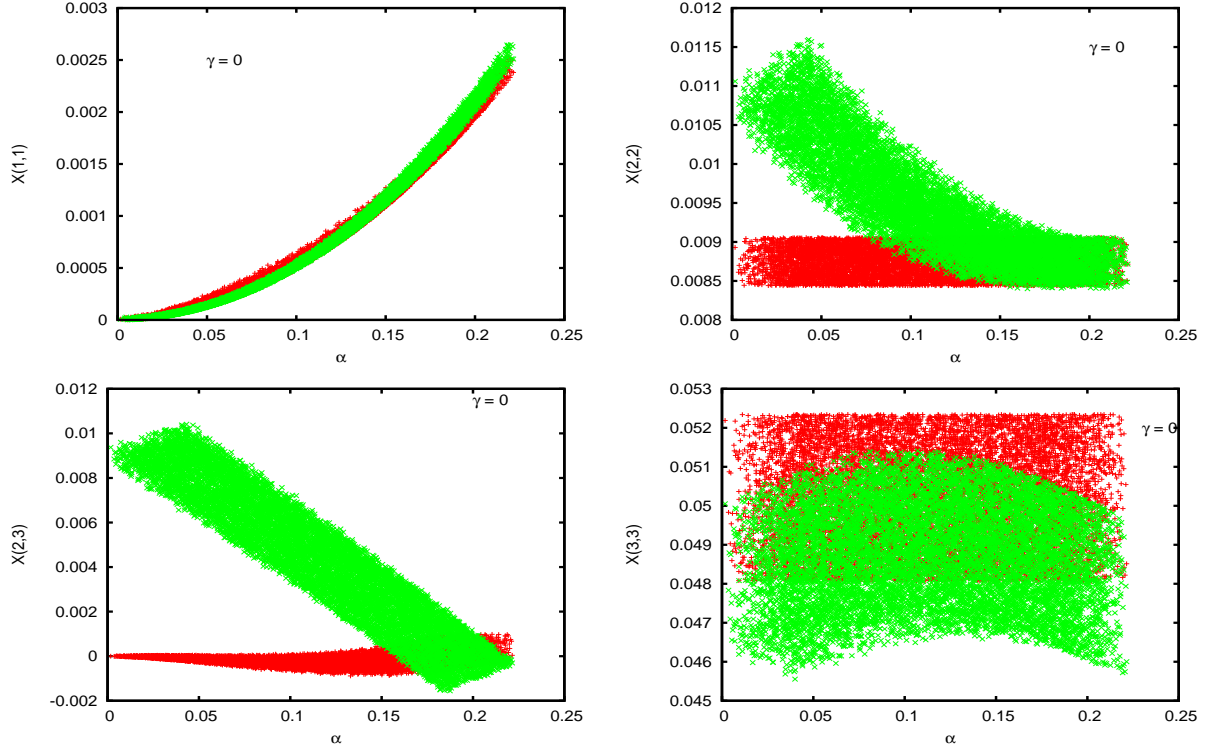


FIG. 6: Range of  $X(i, j)$  as a function of  $\alpha$  with  $\gamma = 0$ . The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

a lot of phenomenological models have been proposed in order to explain the non zero  $\theta_{13}$  where a large non zero value of the third mixing angle  $\theta_{13}$  is generated through a perturbation to these mixing scenarios. In this context, we use a perturbative bottom up approach where the neutrino mass matrix in the flavor basis derived from the current experimental  $3\sigma$  allowed ranges is written as a perturbation around a tree level mass matrix that is determined using the TBM mixing ansatz. However, our analysis is more general and can be applied to any other ansatz which predicts  $\theta_{13} = 0$  at the tree-level.

We know that neutrino mass matrix in the flavor basis does depend on the lightest neutrino mass ( $m_0$ ) in addition to the other oscillation parameters. In this context it is worth mentioning that for  $m_0 \ll 10^{-3}$  eV, the perturbed elements does not depend on the value of  $m_0$ . However, in the opposite limit, where  $m_0 \gg 10^{-3}$  eV, we find that the perturbed elements have exponential dependency on the value of  $m_0$ . We factorise the  $m_0$  dependency of all  $\delta M_\nu(i, j)$  using an exponential parameterization as given in Eq. (13). This helped us in determining the exact dependency of the

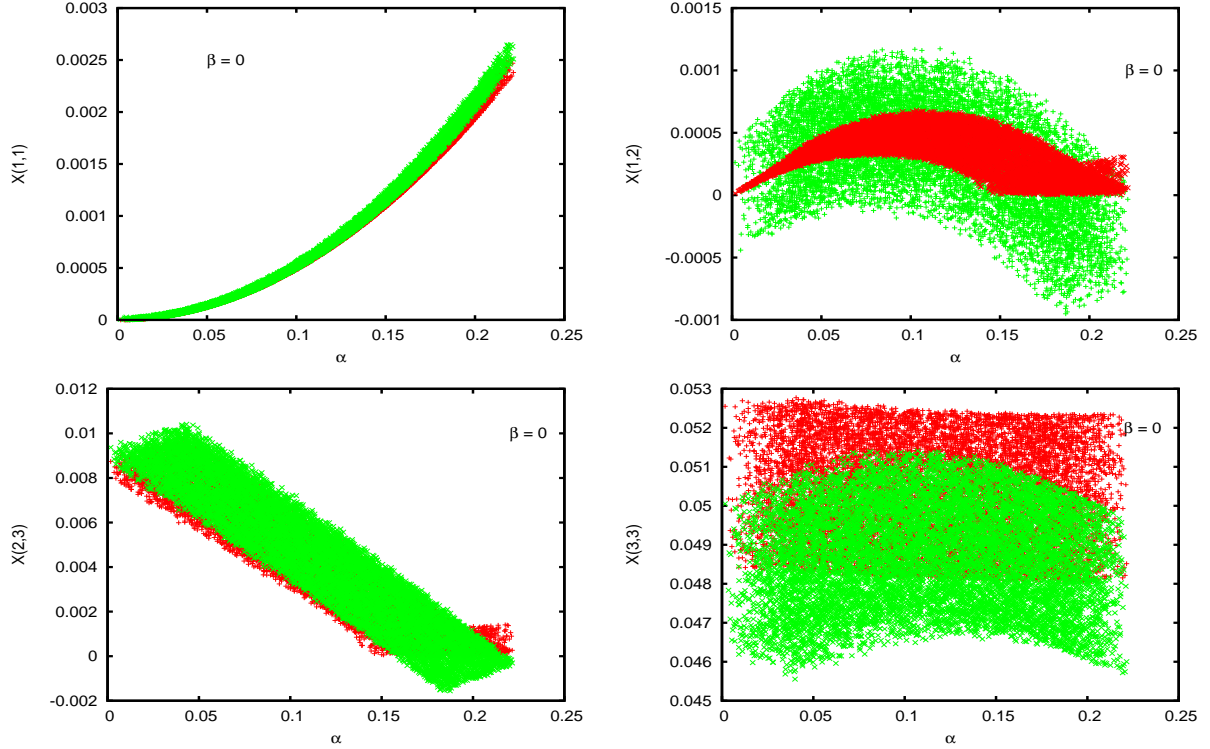


FIG. 7: Range of  $X(i, j)$  as a function of  $\alpha$  with  $\beta = 0$ . The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

perturbed matrix elements on the value of  $\theta_{13}$ ,  $\theta_{23}$  and  $\theta_{12}$ . In order to gauge the size of the perturbation to each element of the mass matrix, we first compared the perturbed matrix elements with the corresponding tree level mass matrix derived from the TBM mixing ansatz and find that only the (1, 2) and (1, 3) elements needs to be modified to be consistent with the experimental data. We also show the exact dependency of the perturbed matrix elements on the value of  $\theta_{13}$ ,  $\theta_{23}$  and  $\theta_{12}$ . We find that, although, (1, 1), (1, 2), and (1, 3) elements of the perturbed matrix do depend on all the three mixing angles, the  $\theta_{12}$  dependency is quite small compared to that of  $\theta_{13}$  and  $\theta_{23}$ . This is, in fact, true for normal as well as inverted hierarchy of the neutrino mass spectrum. However, the (2, 2), (2, 3), and (3, 3) elements depend only on the value of  $\theta_{23}$ . It is expected as the perturbation to (2, 2), (2, 3), and (3, 3) elements of the tree level mass matrix was significantly smaller compared to (1, 1), (1, 2), and (1, 3) elements. Similar conclusions can be drawn for inverted hierarchy as well.

We then use the results of bottom-up approach as guide-lines for determining models of neutrino masses and mixings. We introduce a typical mixing matrix  $U_M = U_{\text{TBM}} V$  for the neutrino mass

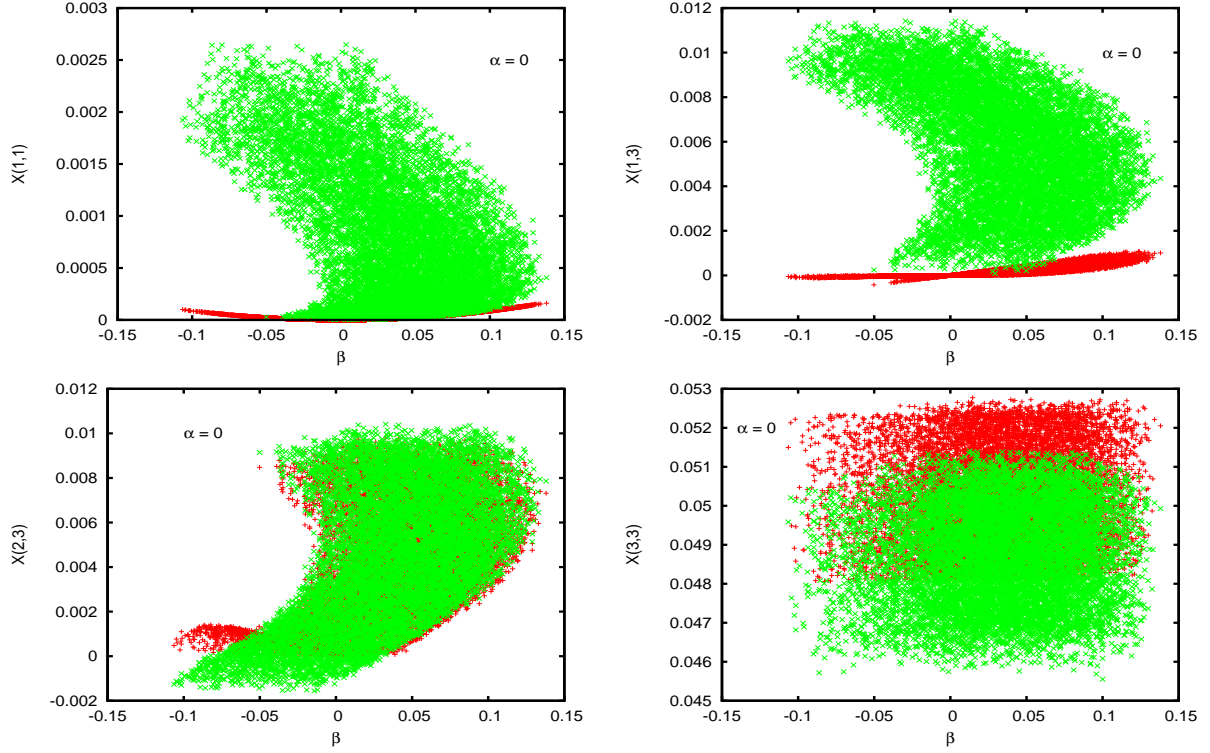


FIG. 8: Range of  $X(i, j)$  as a function of  $\beta$  for  $\alpha = 0$ . The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

matrix, where  $V(\alpha, \beta, \gamma)$  is the perturbed mixing matrix. We fix the parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  from the oscillation data obtained using the bottom up approach. We find that once we set  $\alpha = 0$ , we do not get a good fit with the experimental data that are derived using the bottom up approach. Whereas, we can set  $\beta$  and  $\gamma$  to be zero and still get a reasonably good fit. Since  $\beta$  and  $\gamma$  can be set to zero, their ordering is not important. We further note that, if  $V$  is parameterized by more than one variable then the perturbations are arbitrary. In other words, those can not be treated as true perturbations with respect to their tree level mixing angles.

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